Physics of a Tokamak

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Abstract

Most power plants today rely on nuclear fission, fossil fuels, and renewable sources such as solar and hydro. The Tokamak is an experimental magnetic confinement device designed to produce thermonuclear fusion power. This device was first conceptualized in the 1950s by Soviet physicists, but these days tokamak experiments are global collaborations for example the International Thermonuclear Experimental Reactor.
In this research work, we aimed to derive equations that describe the magnetic fields required to confine plasma, as well as simulate an actual tokamak setup.

Theory

Besides the physics behind plasma formation and fusion, one of the largest difficulties in creating and operating a tokamak is the *confinement* of the plasma with magnetic fields. A classical treatment might equate the thermal energy of the plasma with the kinetic energy and see what magnetic fields are required to contain individual charged atoms, which predicts that:

$$B = \frac{m_e}{Le}\sqrt{\frac{k_B T}{m_e}} \approx_{\text{in S.I. units}} 2 \cdot 10^{-8}\sqrt{T}$$

However, this approach neglects many effects - it is incorrect to treat the plasma as individual particles as they are all interacting with each other. A better model is to consider plasma as a gas/fluid, which uses beta values. Beta values are the ratio between the plasma pressure and the magnetic pressure, and its formula is as follows:

$$\beta = \frac{p}{p_{mag}} = \frac{nk_B T}{B^2/2\mu}$$

In most real life tokamaks, beta is taken to be around 0.01 to adjust for many instabilities that may happen, which then turns our expression solving for magnetic field B to be:

$$B = \sqrt{200\mu_0 n k_B T}$$

Then if we take the n and T to be typical hot plasma values (10^{18} electrons/m^3 and 10^6 K), we get B = 0.6 Tesla, which is much closer in magnitude to the real life tokamak magnetic fields ~ 1 Tesla in orders of magnitude.

Simulation

To create the simulations, we parameterized the wire configuration for a toroid, and split up the toroid into small segments of wires. We then calculated the B-field using the Biot-Savart law. To track movement of particles, we stored the position and velocity of a particle and updated those quantities in small time intervals according to the force acting on it.

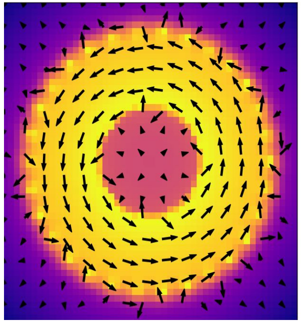

Figure 1. Heat map of the magnetic field strength in a cross-section of a toroidal configuration (brighter colors indicate higher strength). The arrows show the direction of the B-field. In accordance with theory, the B-field is roughly uniform inside the toroid, and negligible outside the toroid.

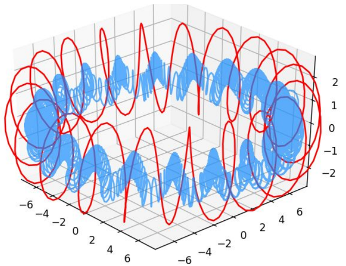

Figure 2. Simulated path of a single charged particle (blue) within a toroidal configuration (red). This shows we can theoretically confine moving particles inside the tokamak, given a strong enough current.

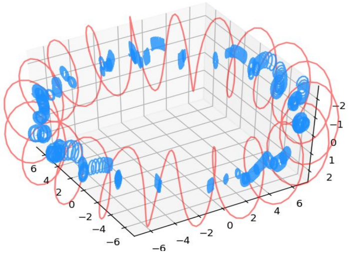

Figure 3. Simulated path of 100 charged particles (blue) within the toroid (red). The particles were initialized with a random position in the toroid and a random velocity, and all of the particles were confined within the toroid, given a strong enough B-field.

Blueprint

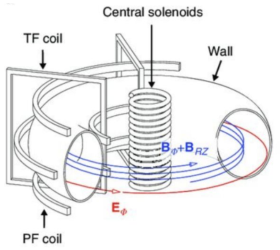

Image 1. A schematic view of the base electrical components of a tokamak, from *Evidence of a turbulent ExB mixing avalanche mechanism of gas breakdown in strongly magnetized systems*, Yoo et al.

The central solenoid coil (CSC) and poloidal field (PF) coils are conductors designed to carry an electrical current circulating in the poloidal direction (in blue, Image 1). This induces a plasma current in the poloidal direction, which, by the Biot-Savart law, tells us that a magnetic field will be created in the toroidal direction, curling about the minor axis. In addition, toroidal field (TF) coils contribute to an electric field in the toroidal direction, which in turn induces a magnetic field in the poloidal direction (again, in blue).[2] The net effect of the magnetic fields would ideally result in field lines which wrap helically around the torus. This contributes twofold towards achieving controlled fusion: first, the effect of ohmic heating, proportional to I^2R, arises[3]; second, the helical form of the **B** field lines theoretically keeps the plasma in a confined region. For reference, the International Thermonuclear Experimental Reactor (ITER) has a current of 46 kA through its CSC.[4]

Image 2. A "tabletop-sized" tokamak, with a major radius of 4 cm. Constructed by Harold Barnard in 2005. Barnard, 2009.

References

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