Relating Electromagnetic Waves to Light

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Introduction

The permittivity and permeability of free space, ε_0 and μ_0, are important physical constants due to their use in relating the strength of the electric and magnetic fields to the medium they permeate. In a similar way that we can relate the properties of a string to the speed of a wave traveling along it, we will use this idea to relate them to the speed of light in a vacuum, c. In this project we seek to determine and relate these constants to the speed of light using a derived setup and simulation.

Background

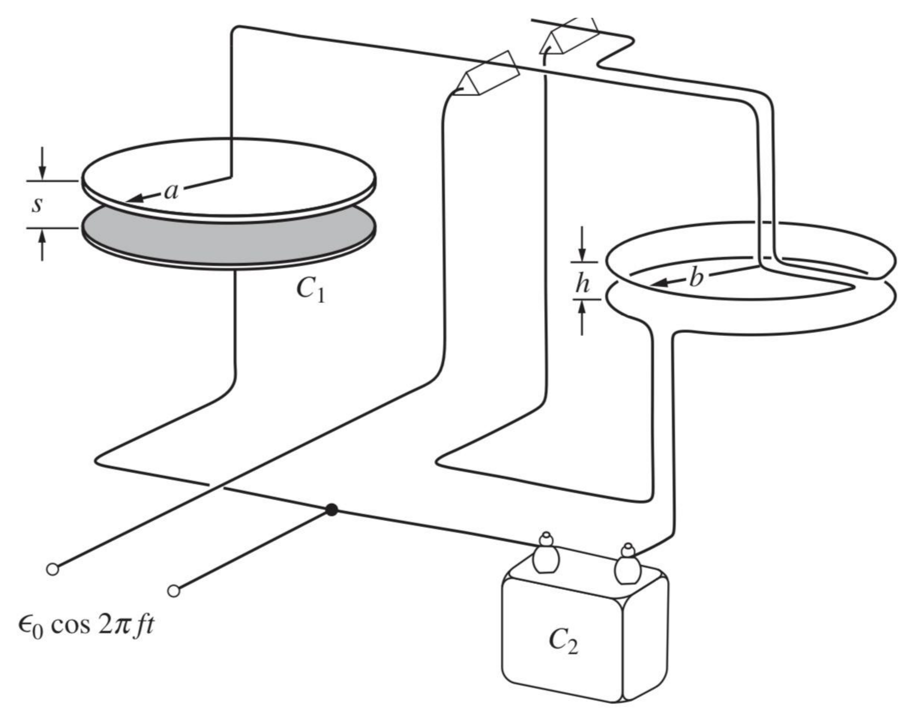
We started by studying a simple wave (a vibrating string) to derive the wave equation and the general form of solutions[2]. Then, by using vector identities and the Laplacian on Maxwell's equations, we showed that the electric and magnetic fields take the form of the wave equation. Simply by comparing coefficients between the general wave equation and the electromagnetic equations, we see that the velocity squared of electromagnetic waves is $1/\mu_0\varepsilon_0$.

$$\frac{1}{v^2}\frac{\partial^2 q}{\partial t^2} = \nabla^2 q$$

$$\nabla^2 \mathbf{E} = \mu_0\varepsilon_0 \frac{\partial^2}{\partial t^2}\mathbf{E} \qquad \nabla^2 \mathbf{B} = \mu_0\varepsilon_0 \frac{\partial^2}{\partial t^2}\mathbf{B}$$

Theory/Set-up

Using the following circuit as a model[3], we can derive the electric and magnetic forces acting on the circuit.



This circuit consists of two capacitors (C1 and C2), and two rings with current traveling in the same direction through them. The circuit is driven at a voltage $\varepsilon_0\cos(2\pi ft)$, where f is the frequency. The entire circuit is set to act as a seesaw that balances when average of the magnetic and electric forces are equal[4]. By applying Gauss' and Ampere's laws, we can arrive at the following expressions for the magnetic and electric forces.

$$\text{Magnetic force} = \mu_0 \tfrac{b}{h}(2\pi f C_2 \epsilon_0)^2 sin^2(2\pi f t)$$

$$\text{Electric force} = \frac{(C_1\epsilon_0)^2 cos^2(2\pi f t)}{2\varepsilon_0 \pi a^2}$$

Equating the average of both forces and solving for $1/\mu_0\varepsilon_0$, we get

$$\frac{1}{\sqrt{\mu_0\epsilon_0}} = (2\pi)^{3/2} a \left(\frac{b}{h}\right)^{1/2} \left(\frac{C_2}{C_1}\right) f,$$

Experimentally, we need to find the right frequency such that the circuit is balanced. This allows us to find $1/\sqrt{\mu_0\varepsilon_0}$ by plugging in our other measured values.

Simulation

We modeled our set-up in Python as a seesaw with an electric force on the left end, identical to that of the parallel plate capacitor, and a magnetic force on the left end, identical to that of the rings of wire. Since, we have functions for each force in terms of frequency and time, we can track the height positions of each end of the seesaw using a bit of calculus and natural initial conditions (we are considering the height of the tip of the fulcrum to be height zero).

We then simply define some position functions of each end that take time and frequency as arguments, plug in any range of time, and test as many frequencies as we want.

In addition to this we also simulated the apparatus using Solidworks to see what the it might look like as it moves. We created a scale-like object and had the electric force pulling down on one end and the magnetic force pulling down the other. We were able to plug in the expressions for them as they varied over time to simulate the forces. This has some drawbacks as the force stays the same even if one end is very far from the initial position, and the force always stays normal to the plates, which is not what would happen in real life. However, we should be able to see approximately what happens at frequencies where it does not oscillate much, and initially what happens at frequencies that are incorrect before the apparatus moves very far.

Results

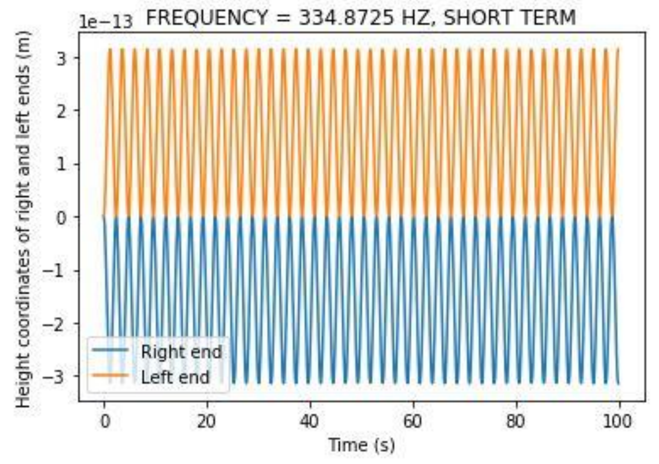


Figure 1. Oscillation of right and left end when the frequency is the one determined by using the previous equation.

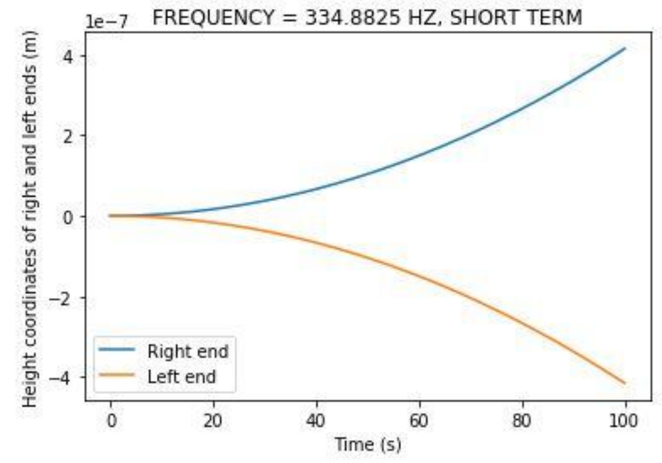


Figure 2. Divergence of right and left ends when the correct frequency is raised by 0.01 Hz.

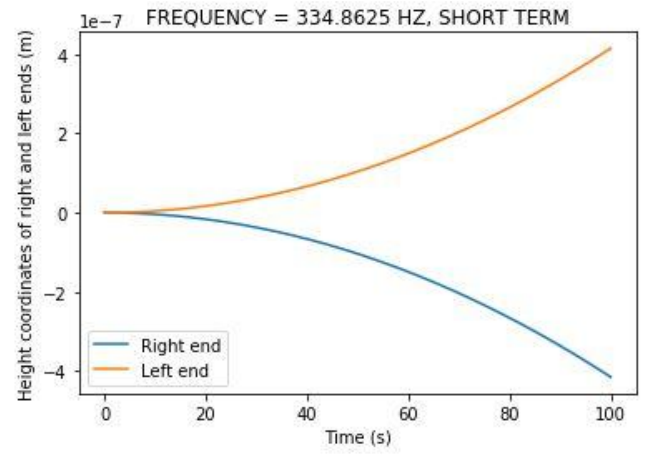


Figure 3. Identical to Figure 2, but the correct frequency is lowered by 0.01 Hz.

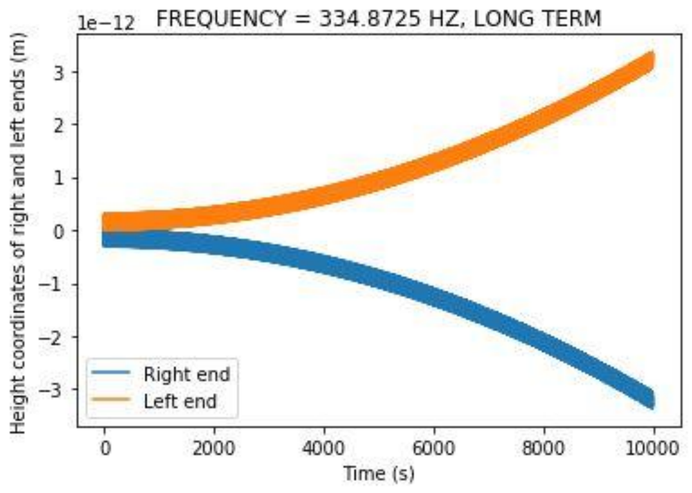


Figure 4. Identical to Figure 1, but this shows long term behavior.

The frequency for Figure 1, is obtained by assuming the following: a = 0.2 m, s = 0.01 m, b/h = 10, the capacitance of the second capacitor is 100 micro-Farads, and the amplitude of the voltage is 10 volts.. These values are obtained by what would have been available to us if we had obtained all the materials from the materials list we made in order to build the system. Looking at the scale of the y-axes on each graph, the correct frequency forces the system to oscillate at a magnitude of 10^{-13} meters, while the "off"-frequencies diverge at a magnitude of 10^{-7} meters. In particular, Figures 1-3 show that the teetering is very sensitive to the frequency of the voltage. Figure 4 shows Figure 1 at a larger time scale, where the oscillation begins to diverge. To explain this, let's graph a couple more things:

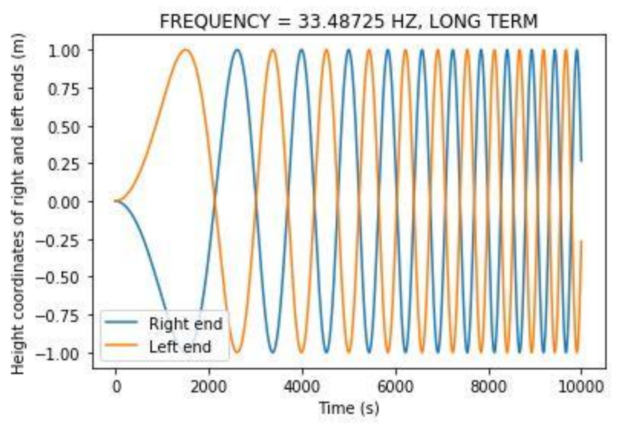


Figure 5. Correct frequency divided by 10, set to a time scale of 1000 seconds.

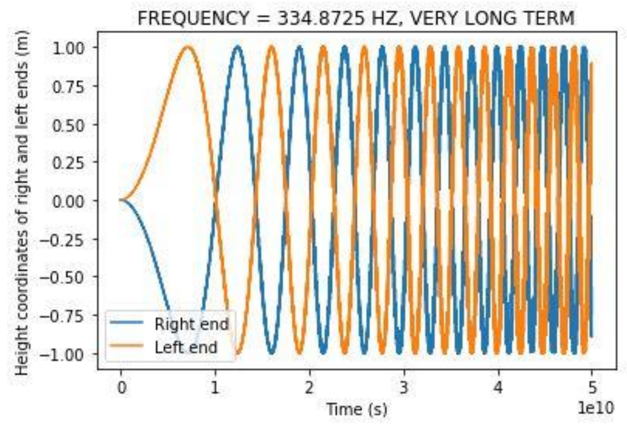


Figure 6. Correct frequency, set to a time scale of about 16 centuries!

This seems to tell us that no matter what frequency we use to alternate the voltage, the ends of the seesaw will always diverge and orbit the pivot faster and faster. The correct frequency seems to maximize the system's resistance to diverge. Here are a few more graphs, whose behavior further supports this conclusion:

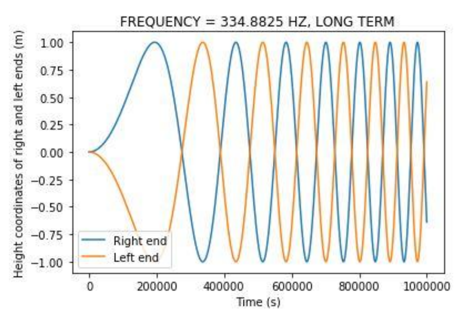


Figure 7. Identical to Figure 2, but time scales to 10^6 seconds.

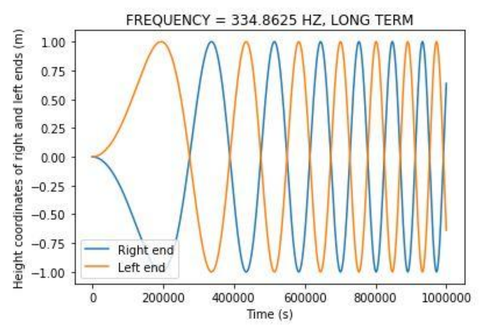


Figure 8. Identical to Figure 3, but time scales to 10^6 seconds.

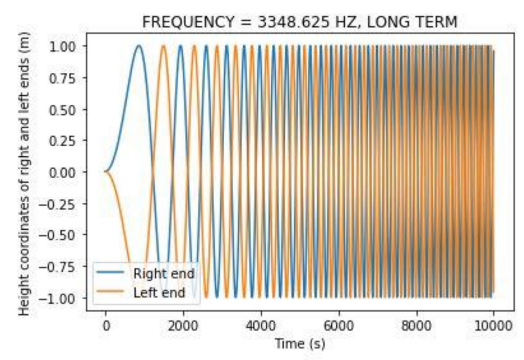


Figure 9. Correct frequency times 10 at a time scale of 10^4 seconds.

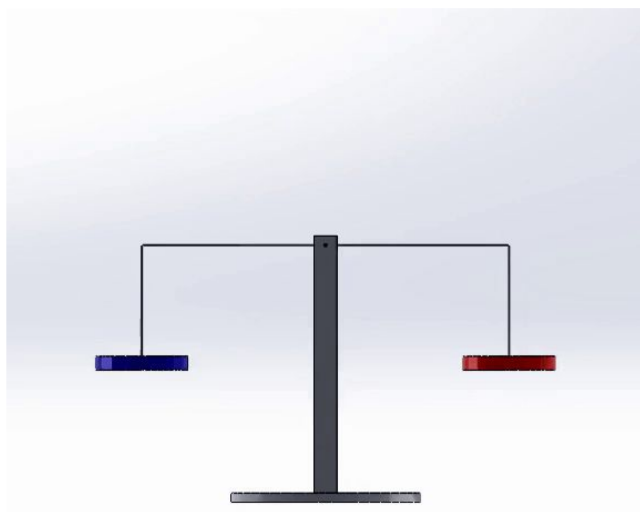


Figure 2-1: Ideal theoretical Frequency. Electric force acts on the blue and magnetic on the red. Forces are exaggerated to illustrate the motion

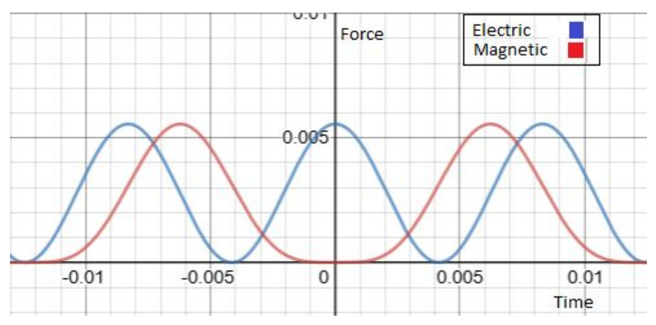


Figure 3-1: Representation of forces. (Not to scale) [5]

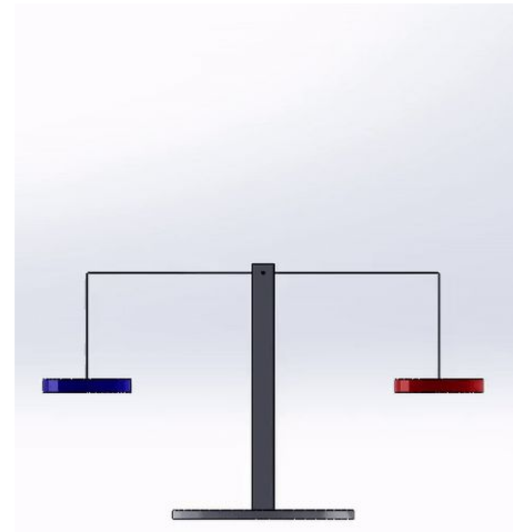


Figure 2-2: 20Hz below ideal frequency. Electric force acts on the blue and magnetic on the red. Forces are exaggerated to illustrate the motion

Figure 3-2: Representation of forces. (Not to scale) [5]

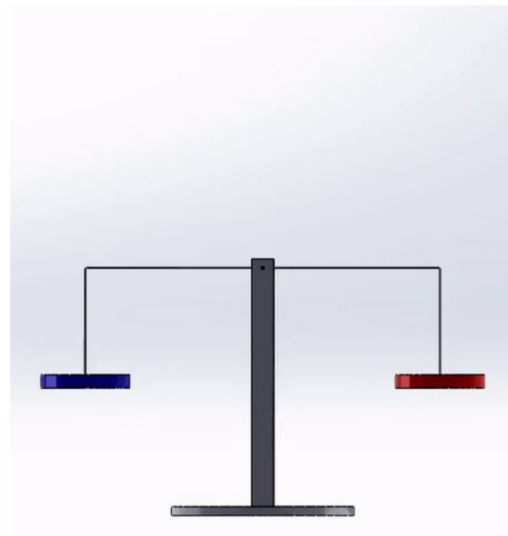


Figure 2-3: 40Hz above the ideal frequency. Electric force acts on the blue and magnetic on the red. Forces are exaggerated to illustrate the motion

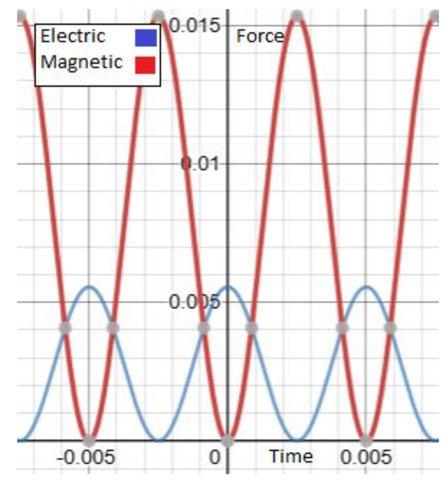


Figure 3-1: Representation of forces. Electric force is blue and magnetic is red. (Not to scale) [5]

In the set up here, the electromagnetic forces constantly act perpendicular to the plates, and to the right a scaled down graph of the forces is shown to give an idea of how the forces oscillate. We can see that at the correct frequency the amplitudes are equal to each other. This causes very small oscillations on the apparatus that balance out. When it is too large, the magnetic force dominates and pulls the red plate down, causing the apparatus to tip over. When it is too small, the electric force dominates causing it to tip the other way. This indicates that if we had built our apparatus as initially intended we would indeed have been able to find the correct frequency by observing how the apparatus oscillates. It should be noted that in this simulation a much higher voltage than is realistic was used so that the oscillations were visible. In reality, it would likely appear still.

Limitations of our Experiment

We initially anticipated this project to be primarily experimental, as it involves a physical circuit model from which we derive our theory and data. In light of the recent global pandemic and cancellation of all in-person activities on the Berkeley campus, we were not able to finish setting up and collecting data from the apparatus. We decided to do a theory-based project in which we represent our apparatus with simulations and graphs using expected values relevant to the calculation of the forces. However, the capability of our simulation falls short of that of the physical apparatus. In the Solidworks simulation the forces were exaggerated to illustrate the motion by using a very high voltage, so it would harder to spot the correct frequency in real life. In addition, it does not completely capture the motion as the further the plates or rings are from each other, the smaller the attractive force between them. This is why the simulation shows the apparatus spinning rather than merely tipping, in addition to the fact that they stay normal to the plate.

Future Work

As we were unable to actually conduct the physical experiment, a future project could be building the theoretical set-up. From this, we could match the experimental value for the frequency vs. the simulation value.

Another theoretical result we found was that if we increased the number of turns in the inductor (N), the balancing frequency would decrease by N. This could be used to study the sensitivity of the circuit as well.

References

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