Doppler Imaging of a Simulated Star

Andrew Goh, Andris Huang, *Che Liu, †Yi Zhu
University of California Berkeley, Department of Physics and Astronomy, Undergraduate Lab at Berkeley
*Mentor †Lab Director

Motivation

- Simulate a star with starspot features and create an artificial surface brightness map
- Calculate a simplified, hypothetical spectra assuming the star radiates like a perfect blackbody
- Attempt to recover the surface brightness map from the hypothetical spectra using the maximum entropy method

Background and Assumptions

- **I** vector (image vector): A 1D array containing the surface brightness values for each surface element of our simulated star
- **D** vector (data vector / line spectrum): An 1D array containing flux values at different wavelengths
- **R** matrix (transfer matrix): A 2D array that is used to calculate D via the matrix equation **IR = D**
- We did not take in any accounts for limb darkening, atmospheric radiative transfer or chemical compositions
- Simulated star spots are circular, completely arbitrary and non-physical
- The star's inclination (i) is towards the line of sight

Constructing the I vector (Building the Star)

- The simulated star is initialized with a number of surface elements, stellar radius, equatorial velocity, inclination angle, and background temperature.
- Longitudinal and latitudinal information is calculated for each surface element of the star.
- Artificial star spots can then be generated at any location given a longitude, latitude, radius, and temperature.

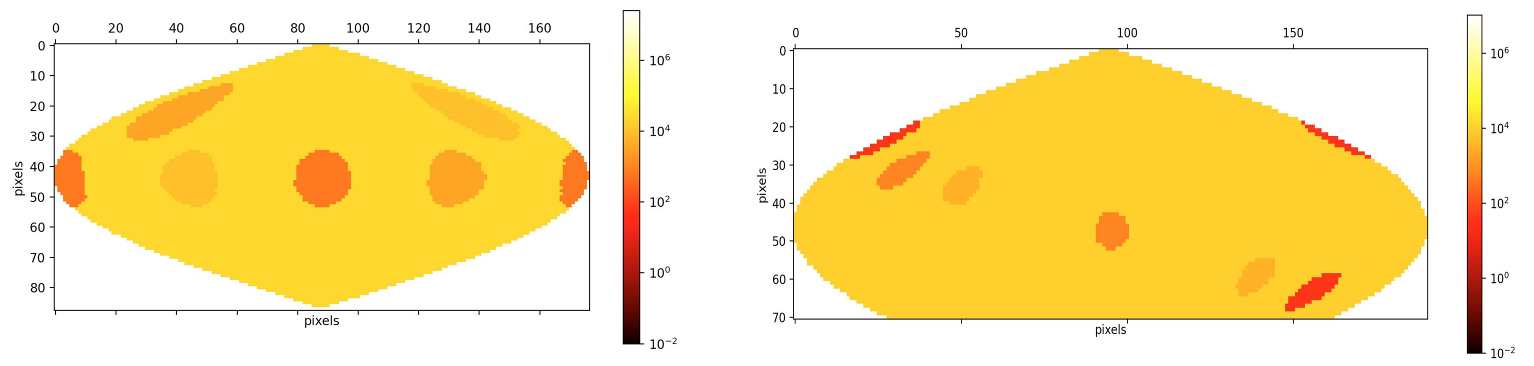

Fig 1. A surface temperature map for two different example test stars with ~1000 surface elements. Features farther away from the center appear to be distorted due to our configuration of the map being a stack of latitudinal disks. The image on the right corresponds to a star with an inclination angle of 42.86 degrees, rendering the bottom section of the star invisible to the observer.

Constructing the R matrix and calculating D (Forward Method)

- The star will radiate like a blackbody, with spots varying in temperature contributing different blackbody curves to the overall line spectra
- Each surface element has a temperature/surface brightness, in which we could calculate its radiation flux from the following equation:

$$\int_{\lambda_1}^{\lambda_2} \frac{2hc^2}{\lambda^5\left(\exp(\frac{hc}{\lambda k_B T})-1\right)} d\lambda$$

- Each surface element has a projectional area towards the line of sight, which acts as a fractional scaling factor towards its flux contribution, calculated by

$$\sin(\theta)\sin(\phi)\sin(i) + \cos(\theta)\cos(i)$$

- Each surface element has a radial velocity (towards line of sight) that contributes to a doppler shift of the spectrum, calculated by using the PyAstronomy package.

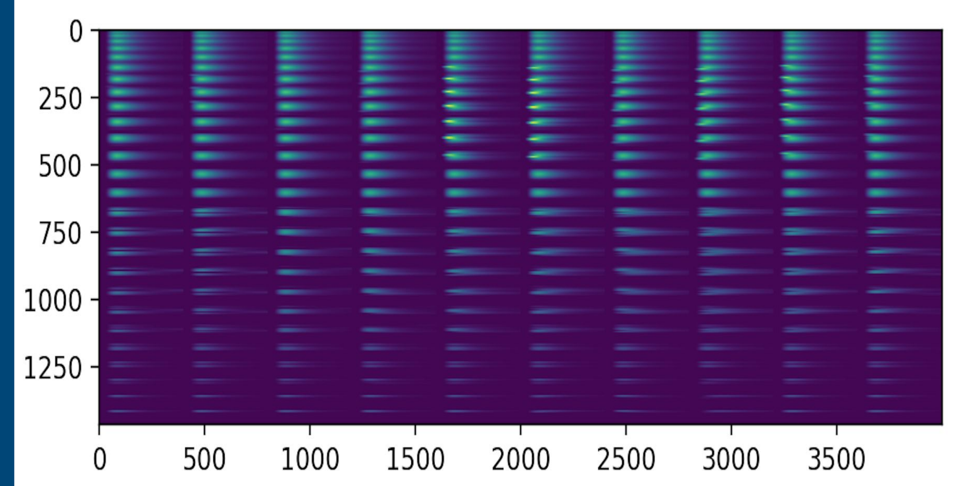

Fig 2. Visual representation of the R transfer matrix for a high pixel count test star. The y axis corresponds to pixels in the image vector, and the x axis represents wavelength. The gradient color intensity represents the intensity of the scaling factor. The apparent slices represent different rotational phases, in which we decided to work with 10 rotational phases.

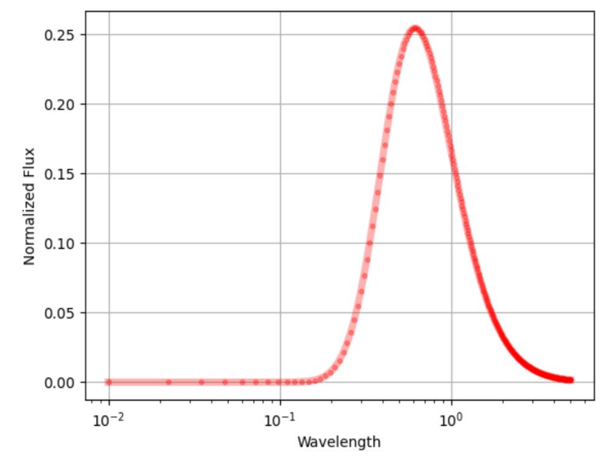

Fig 3. A calculated spectra at a certain rotational phase. Units are arbitrary and the wavelength is on a log scale. Our calculated spectra looks trivially like a blackbody curve, but the spectrum is observed to predictably change in intensity and shift at very small wavelengths at different rotational phases.

Recovering the I vector (Inverse Method)

- Given the **R** matrix and Data vector, we are able to recover the **I** vector in two ways
  - Pseudo Inverse (Least Squares Solution)
    - Since the matrix **R** is not invertible, we'll use the pseudo inverse of R to estimate the image vector using the NumPy pseudo inverse function

$$R^{+}I = D$$

  - Maximum Entropy Method
    - Searches for the **I** vector that maximizes

$$Q = S - \lambda\chi^2$$

where λ is a scaling constant factor that requires manual fine-tuning, and

$$\chi^2 = \sum_k (g_k - d_k)^2/\sigma^2$$

g is the individual elements in the computed **D** vector, d is the element in true data vector **D**, and σ = 1.

$$S = -\sum_j p_j \log(p_j)$$

S is the Shannon entropy where p is the normalized surface brightness of an individual element in the guess **I** vector.

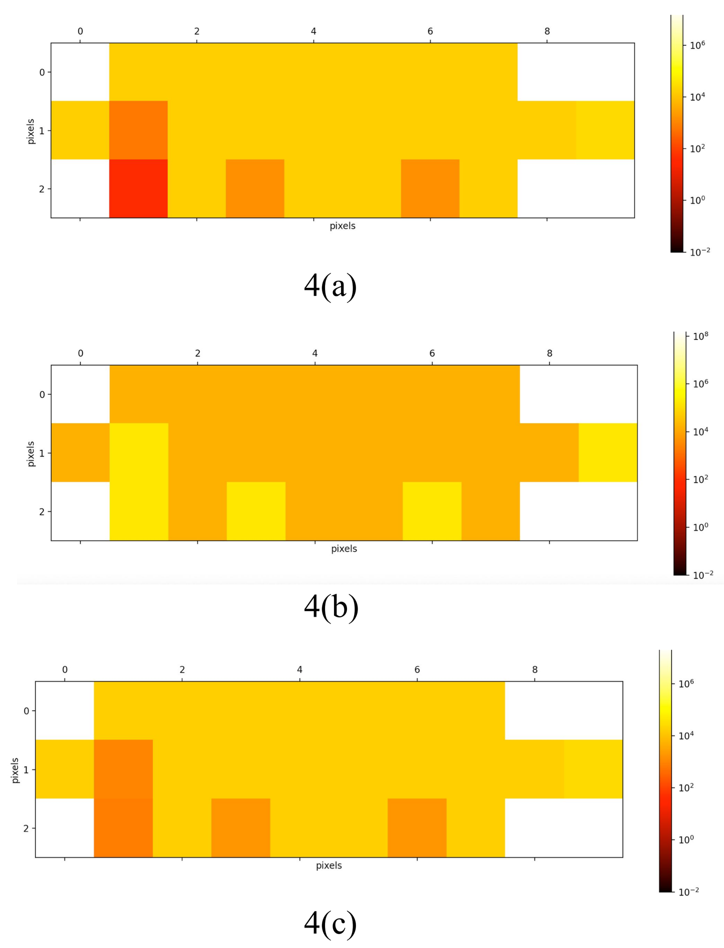

Fig 4. A demonstration of the maximum entropy method with low resolution stars of ~24 surface elements. 4(a) is the "ground truth" test star. 4(b) is a "guess" star with wrong temperature spots. The image vector of the "guess" star serves as an input for the maximum entropy method, and in its searching process, outputs 4(c), with corrected temperatures.

Results

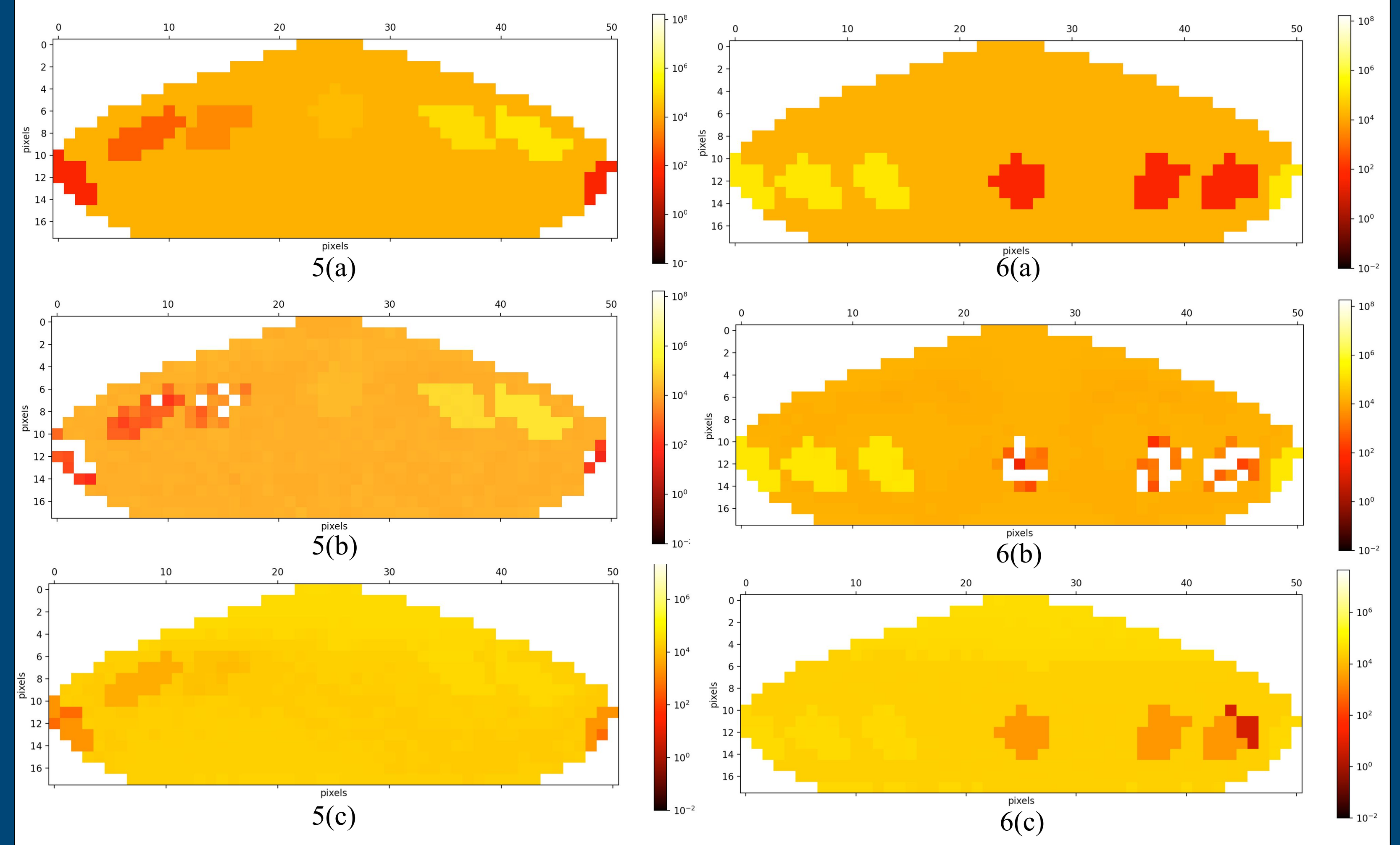

Fig 5 & 6. Shows the surface brightness map of two different test stars of ~700 surface elements 5(a), 6(a) and the recovered surface brightness map using the pseudo inverse method 5(b), 6(b) and the maximum entropy method 5(c), 6(c). The inclination angle is 42.86 degrees. (Color map scales according to relative temperatures)

- Comparison
  - The Pseudo Inverse Method seems to achieve more accurate temperatures than the Maximum Entropy Method on a high level of resolution, but there are a few unsolvable NaN values produced in the resulting vector for low temperature spots.
  - Maximum Entropy Method managed to recover the spot locations and their relative temperatures given a spotless guess star and a specific λ value, but their temperature values are difficult to distinguish against the background temperature
  - Both of the methods behaved poorly with added artificial noise to the Data vector

Challenges and Future Directions

- Our current inverse method requires knowledge of the temperature map to construct the **R** matrix.
- Our current method uses the test star to calculate the **R** matrix, but this would not be applicable in real scenarios where we would not have information at all about the target star's temperature map. It would also require huge memory space and computational power if we repeatedly construct **R** based on different trials of guess stars.
- For future works, we need to construct **R** such that it does not depend on the temperature map of the true star
- We would also need a faster searching algorithm that can compute the resulting vector in a more efficient and accurate manner

Acknowledgement

We would like to thank the ULAB program for the generous support. We would also like to thank our mentor Che Liu for his patient guidance throughout this project.

References

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