Analyzing Anomalous Transport in Interplanetary Shocks Using a Mittag-Leffler Function

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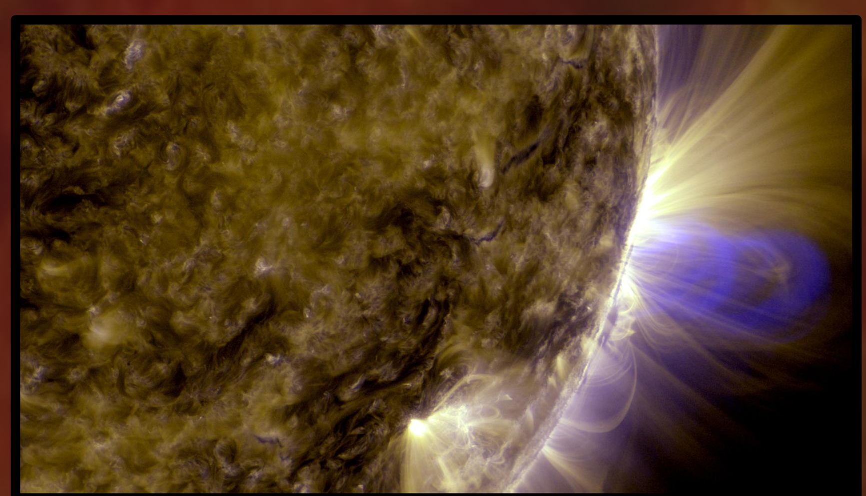
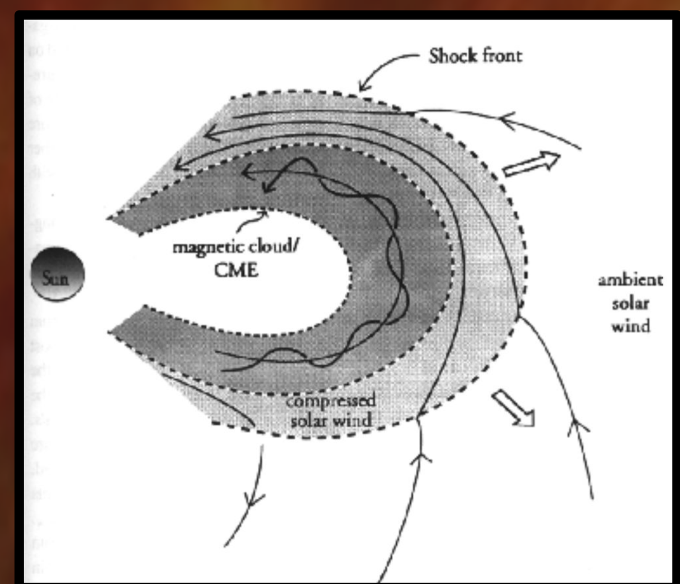
Background

Coronal Mass Ejections (CMEs) are large heliospheric transients that result from entangled magnetic fluxes undergoing magnetic reconnection, a process by which magnetic fields attempt to realign themselves in order to achieve lesser tense configurations. The process ejects solar matter and its accompanying magnetic energy at incredibly fast speeds, the latter of which is converted to thermal energy, kinetic energy, and accelerates particles. When the speeds of the CME are fast enough, the CME's propagation supersedes ambient solar winds, creating a compressional region bounded by a forward shock, subsequently driving what are referred to as collisionless interplanetary shocks. This process energizes particles through electromagnetic acceleration[1].

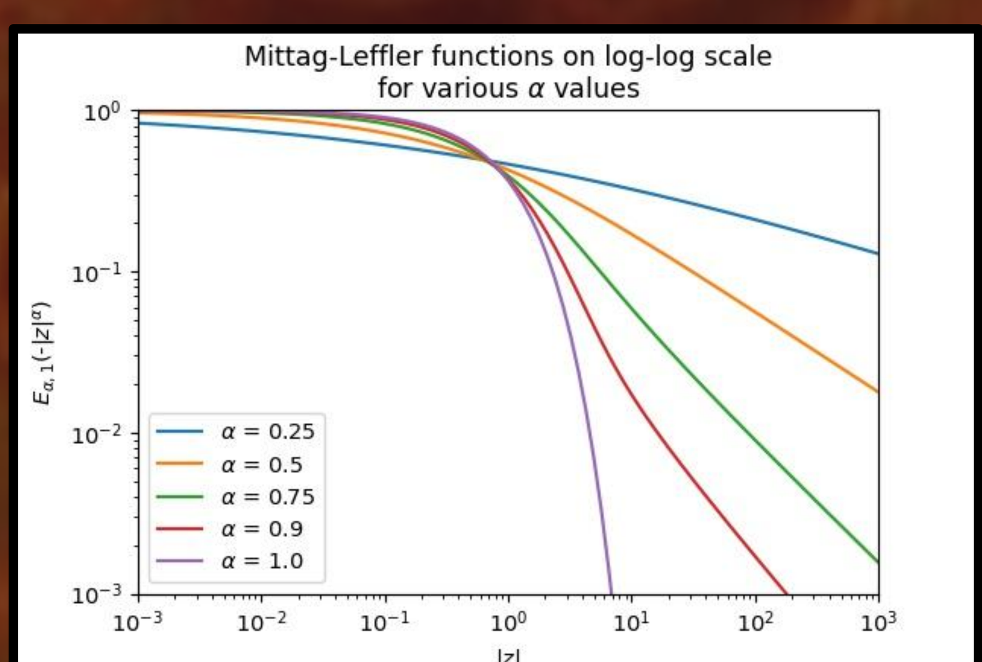




Although research of energetic particle behavior after these collisionless interplanetary shocks is relatively perfuse, the nature of particles ahead of them remains limited despite its significance to space weather phenomenon and understanding energetic particle transport. The process of shock acceleration is influenced by the motion of charged particles both upstream and downstream the shock front, but data has offered evidence of superdiffusive transport—a type of anomalous transport, also commonly referred to as anomalous diffusion, which characterizes any diffusion that describes a non-linear relationship between time and the mean square displacement—upstream of collisionless interplanetary shocks in the heliosphere[5], suggesting that anomalous particle transport is driven by electromagnetic interactions. In recent research, a Mittag-Leffler function has been used to model upstream superdiffusive particle behavior with various parameters, though in need of further exploration.

Mittag-Leffler

$$J_T(x) = J_T(0)\sum_{k=0}^{\infty}\left(\frac{(-(\frac{x}{L})^{\alpha-1})^k}{\Gamma((\alpha-1)k+1}\right)$$



The ordinary and generalized Mittag-Leffler functions interpolate between a purely exponential law and power-like behavior governed by ordinary kinetic equations and their fractional counterparts.

Since the data of energy flux versus time that we are analyzing for our shock events correspond to similar exponential laws and power laws, a Mittag-Leffler plot would give us the necessary insight to understand the anomalous transport of the energetic particles.

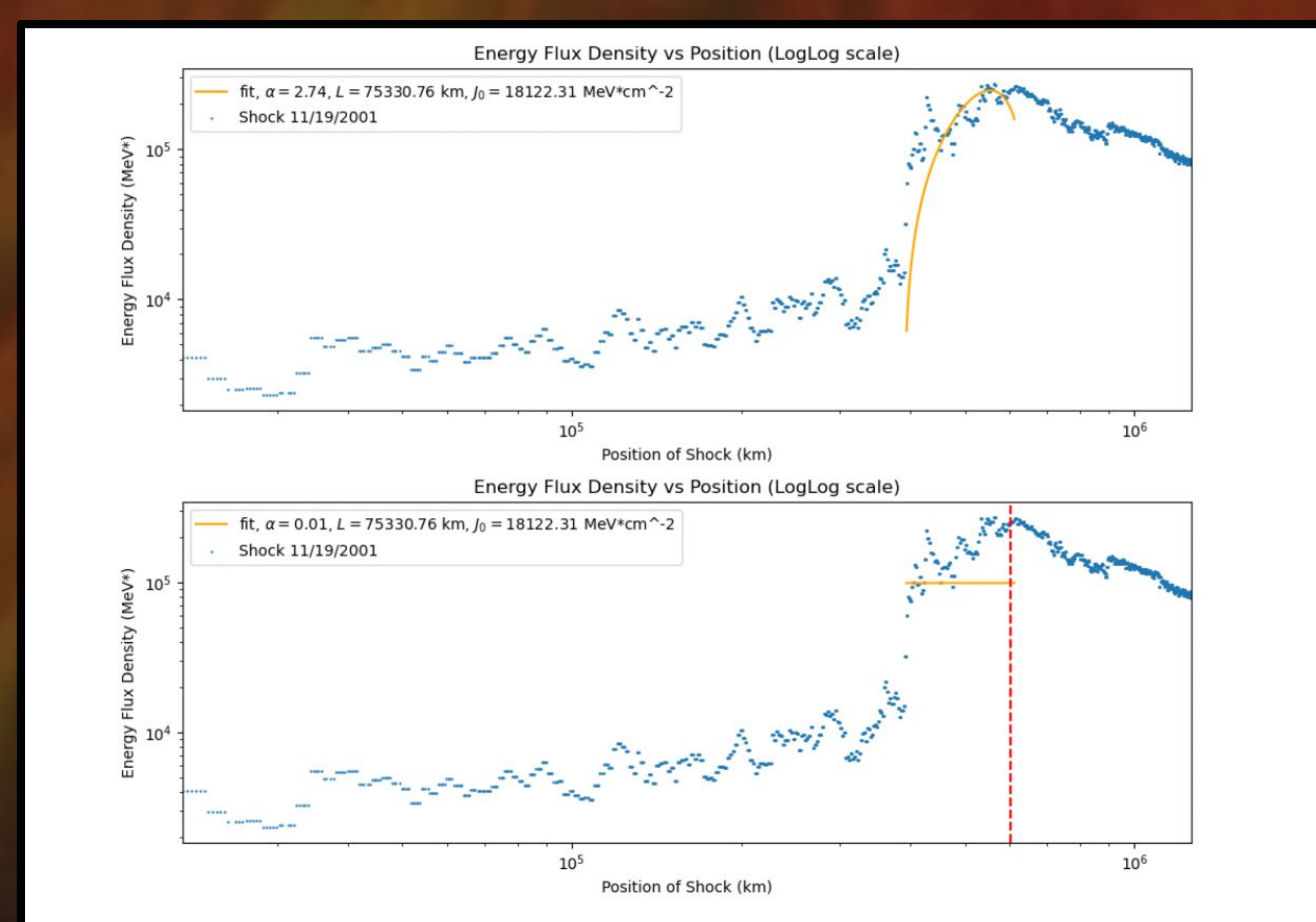
Methods

We will be worked primarily with data collected by the Electron, Proton, and Alpha Monitor (EPAM) and the Solar Wind, Electron, Proton, and Alpha Monitor (SWEPAM), two instruments aboard the Advance Composition Explorer (ACE) spacecraft. EPAM measures the number of incoming particles and is capable of separating them by energy. SWEPAM is capable of measuring proton density, the radial component of proton temperature, the ratio of alpha density to proton density, proton speed, proton velocity in GSE, RTN, and GSM coordinates, and electron temperature.

We used the EPAM data to plot energy flux at given points in time. The time of CMEs and their related interplanetary shocks are recorded in the Database of Heliospheric Shock Waves maintained at University of Helsinki (also known as IPshocks).

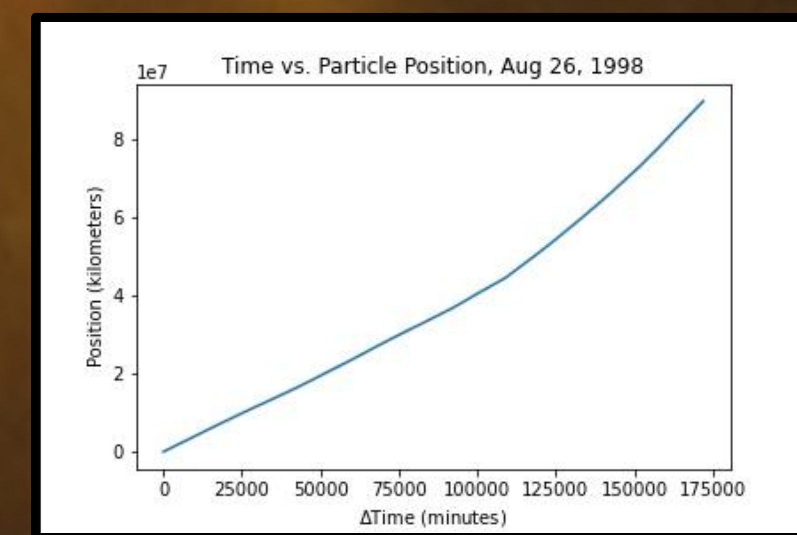
Results

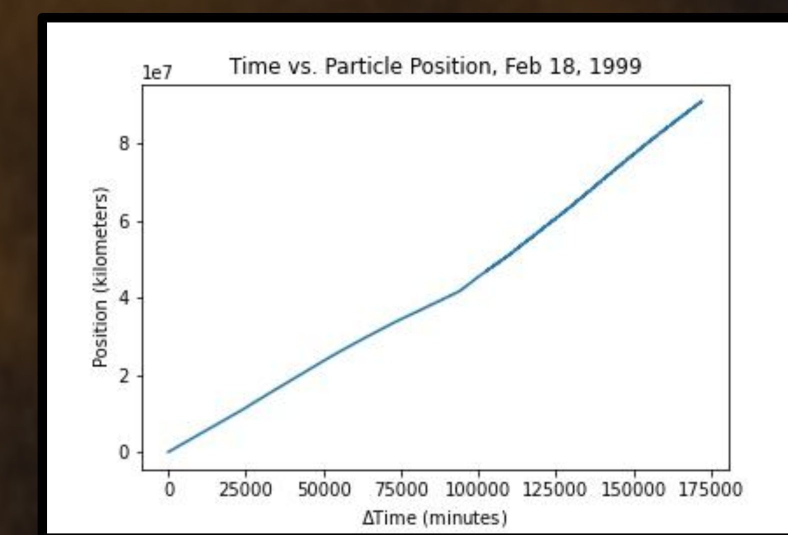
Zimbardo's original fit was against a time versus energy flux plot—a decision made in virtue of the fact that Zimbardo and his team designated a constant value for velocity[5]. Subsequently, energetic particles upstream the shock could be modeled with a linear position through time. However, contrary to Zimbardo's methodology, the energetic particle velocity is not constant and fluctuates through time. In light of this, we modified the data by integrating the velocity as a function of time in order to acquire the position (right), allowing us to instead use position for our x-axis since, as stated before, the position would no longer be linearly dependent on time. With everything considered, our results repeatedly showed incongruities when compared to Zimbardo's findings. His alpha values were in the range between one and two for proper fits, but for us to achieve an adequate fit of the Mittag-Leffler function similar to his, the alpha values had to consistently exceed two (2.74 for the 2001 November 19 shock event). Trusting the theory, it is physically impossible for alpha to be of that value. Moreover, trying to best fit within the theoretical bounds of alpha gives a horizontal line: a clearly erroneous result. All considered, we conclude that the Mittag-Leffler function is not an optimal function for modeling energetic particle behavior upstream of collisionless interplanetary shocks.



Analysis of Velocity Incorporation

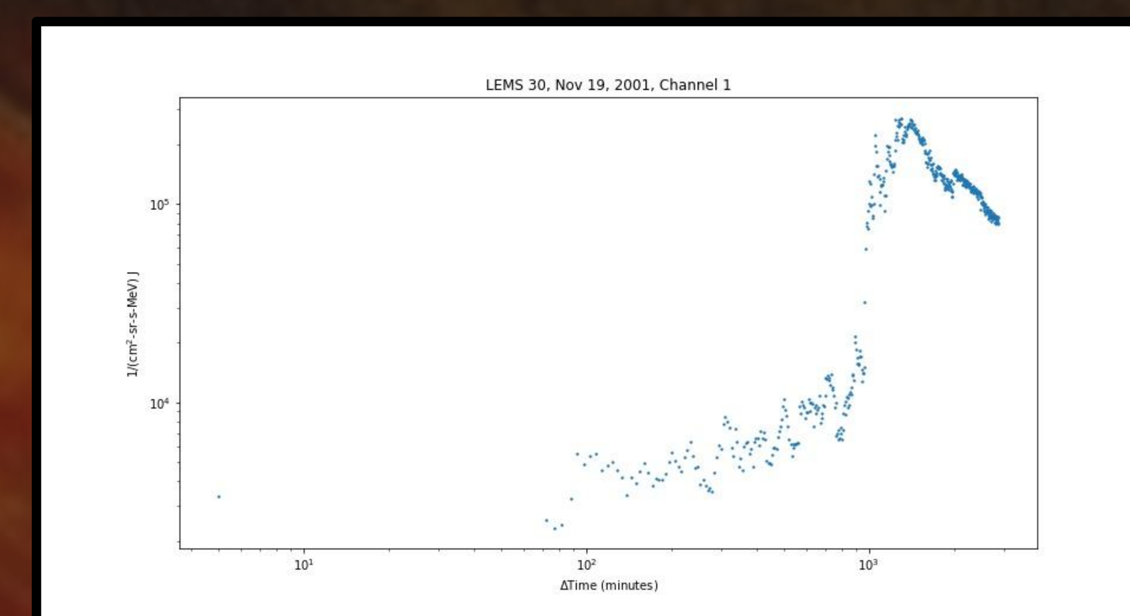
Previous analysis of shocks[2,4] utilize an average velocity to convert from time to position. Using particle velocity data from SWEPAM, we were able to use Simpson's integration method to calculate a more accurate estimate of position. The following is a example of this conversion between time and position, which clearly displays non-linear behavior.





Data

The data we collected was from both the the Electron, Proton, and Alpha Monitor (EPAM) and the Solar Wind, Electron, Proton, and Alpha Monitor (SWEPAM), two instruments aboard the Advanced Composition Explorer (ACE) spacecraft. EPAM measured the number of incoming particles and was capable of separating them by energy. The time of coronal mass ejections and their related interplanetary shocks are recorded in the Database of Heliospheric Shock Waves maintained at University of Helsinki (also known as IPshocks). We mainly focused on readings near those times.



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References

1. Emilia Kilpua, Hannu EJ Koskinen and Tuija I Pulkkinen. Coronal mass ejections and their sheath regions in interplanetary space. Living Reviews in Solar Physics, 14(1):1-83, 2017.
2. S Perri and G Zimbardo. Superdiffusive shock acceleration. The Astrophysics Journal, 750(2):87, 2012.
3. G Prete, S Perri, and G Zimbardo. Energetic particle fluxes at heliospheric shocks: Evidences of superdiffusion and comparison between analytical and numerical modeling. New Astronomy, 87: 101605, 2021.
4. Gaetano Zimbardo and Silvia Perri. Non-markovian pitch-angle scattering as the origin of particle superdiffusion parallel to t he magnetic field. The Astrophysical Journal, 903(2):105, 2020.
5. Gaetano Zimbardo, Giuseppe Prete, and Silvia Perri. Collisionless shocks as a diagnostic tool for understanding energetic particle transport in space plasmas. Frontiers in Astronomy and Space Sciences, 7:16, 2020.